Artificial compressibility for smoothed particle hydrodynamics using pressure smoothing

Joe J. De Courcy, Thomas C.S. Rendall,

Brano Titurus, Lucian Constantin and Jonathan E. Cooper

University of Bristol, U.K.

Military Technical Academy “Ferdinand I”, Romania.

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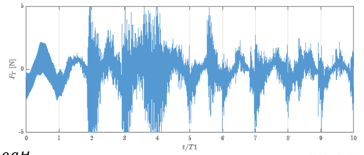
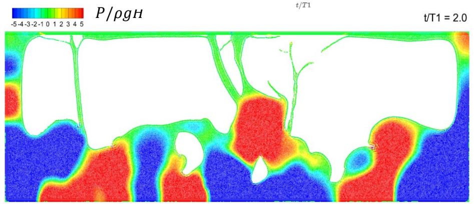
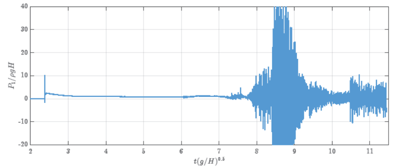
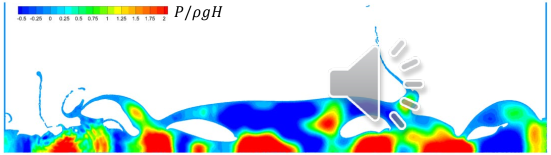
Introduction

- Motivation
  - Modelling sloshing fuel in aircraft wings
  - Residual acoustic waves in WCSPH interacting with structural model
- Objectives
  1) Develop a fully incompressible solution using artificial compressibility (ACSPH) with a focus on FSI solutions
  2) Discuss and highlight the links between WCSPH, ISPH and ACSPH (accidental!)
- Success?
  1) Yes, ACSPH improved predictions
  2) WCSPH (δ-SPH), ISPH and ACSPH are much closer relatives than we thought at the beginning – but perhaps everyone else knew this already?









Background

Ignoring fully coupled methods such as **Xiao JCP 2017**

- Chorin '68, pressure Poisson eq.

$$\frac{Du}{Dt} = -\frac{\nabla p}{\rho}$$

Take divergence

$$\nabla \cdot \frac{Du}{Dt} = -\rho \nabla^2 p$$

Ideally zero

Form pressure perturbation eqn.

Wide number of variants – **Guermond, CMAME 2005**

$$\nabla \cdot \frac{D(u^{i+1,k+1} - u^{i+1,k})}{Dt} = -\rho \nabla^2 (p^{i+1,k+1} - p^{i+1,k})$$

Force next velocity to be solenoidal

Many variants available…

$$\frac{Du^{i+1,k}}{Dt} = -\frac{\nabla p^{i+1,k}}{\rho}$$

$$-\nabla \cdot \frac{Du^{i+1,k}}{Dt} = -\rho \nabla^2 (p^{i+1,k+1} - p^{i+1,k})$$

Normally this isn't looped in i, but it has been tried, and works well – **Aoussou, JCP 2018**

I don't see these methods as different

- Chorin '67, artificial compressibility

$$\nabla \cdot u = 0 \rightarrow \frac{Dp}{D\tau} = -k \nabla \cdot u$$

Use pseudo-time

$$\frac{Dp^{i+1,k+1}}{D\tau} = -k_1 \nabla \cdot u^{i+1,k} - k_2 \nabla \cdot \left(\frac{Du^{i+1,k}}{Dt} + \frac{\nabla p^{i+1,k}}{\rho} \right) + \cdots +$$

Similar to above. Could we put some more error forcing on the RHS? **Yes, we will test 4th differences similar to JST later**

This form is just another version of the energy equation, which is why it can also be used explicitly in real time if compressible, or weakly compressible

…or, can be used in pseudo for exact incompressibility, which is AC

$$\frac{Dp}{Dt} = -\gamma p \nabla \cdot u + \frac{\gamma \upsilon}{P_r} \nabla \cdot \nabla p$$

Entropically damped artificial compressibility or 'EDAC', Clausen, Phys. Rev 2013

EDAC SPH Ramachandran, CAF 2019

Dual time - Ramachandran, CAF 2021

Putting the Laplacian part on the LHS requires linear solves or pseudo time

Digestion

- Incompressible mesh based methods often use PPE; AC less common
- AC meshed methods *hardly ever* use the Laplacian term (some exceptions). Presumably, this is due to smoother mesh based pressure fields, and the originally proposed formulation

Toutaint, Phys. Letters 2017

Dupuy, JCP 2020

- AC for SPH has been tried but rarely including Laplacian terms (recently more so, however)

EDAC SPH Ramachandran, CAF 2019

- AC was originally for steady flows, but works for unsteady via a pseudo-time loop, as here

Dual time - Ramachandran, CAF 2021

Can this be linked to δ-SPH?

- Yes, because pressure/density are related by an artificial equation of state (linear or nonlinear), thus the link to the Laplacian-type term in density is broadly equivalent to the compressible energy equation
- This implies any δ-SPH implementation can be put in a pseudo-time loop to give exact incompressibility (which was known already)
- Doing this maps to using artificial compressibility with inclusion of the Laplacian term
- At discrete/code level, exact equivalence doesn’t hold, of course

$$\frac{Dp}{Dt} = -\gamma p \nabla \cdot u + \frac{\gamma \upsilon}{P_r} \nabla \cdot \nabla p$$

$$\frac{Dp^{i+1,k+1}}{D\tau} = -k_1 \nabla \cdot u^{i+1,k} - k_2 \nabla \cdot \left(\frac{Du^{i+1,k}}{Dt} + \frac{\nabla p^{i+1,k}}{\rho} \right) + \cdots +$$

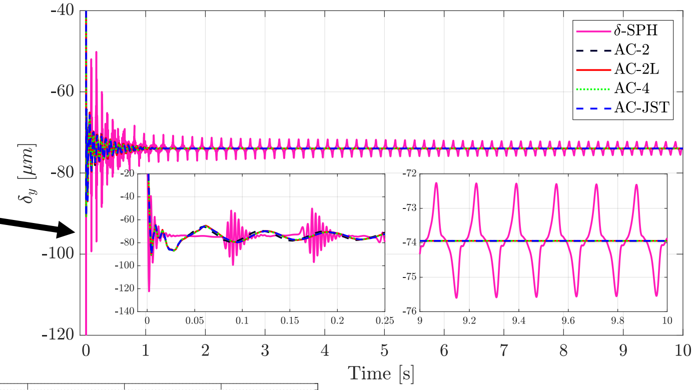
$$\psi_{ij} = 2(\rho_j - \rho_i) \frac{\mathbf{r}_{ji}}{|\mathbf{r}_{ij}|^2} - \left[\langle \nabla \rho \rangle_i^L + \langle \nabla \rho \rangle_j^L \right]$$

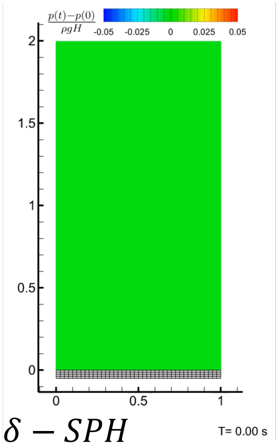
$$\frac{D\rho_i}{Dt} = -\rho_i \sum_j (\mathbf{u}_j - \mathbf{u}_i) \cdot \nabla_i W(\mathbf{r}_j) V_j + \delta h c_0 \sum_j \psi_{ij} \cdot \nabla_i W(\mathbf{r}_j) V_j$$

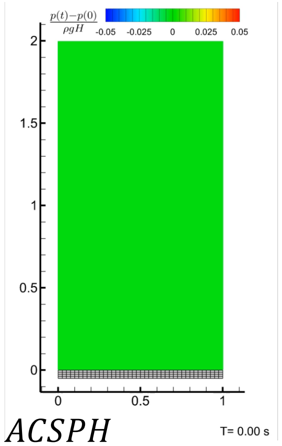
$$p = c_0^2 (\rho - \rho_0)$$

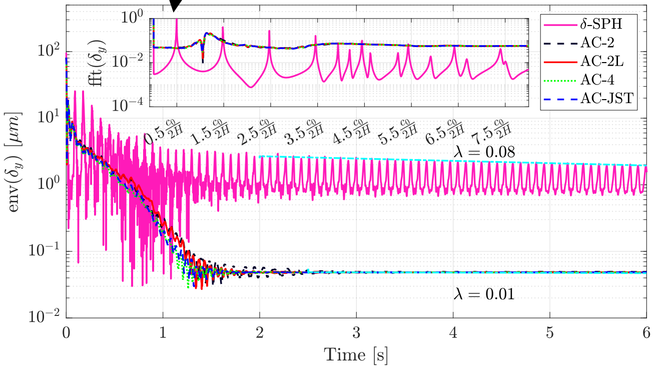
Case 1: Hydrostatic/elastic column

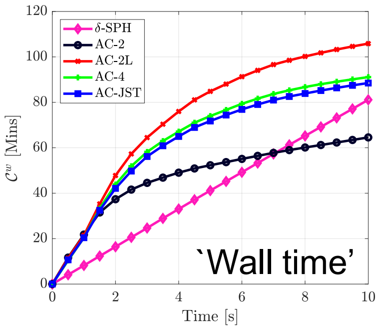
- Hydrostatic column of water is released onto an elastic plate, recording structural displacements
- Acoustic waves in WCSPH interfere with structural response, adding complex amplitude and frequency components into coupled response
- Frequency response dominated by multiples of c_0, dominant component at $\sim 0.5 c_0 / 2H$





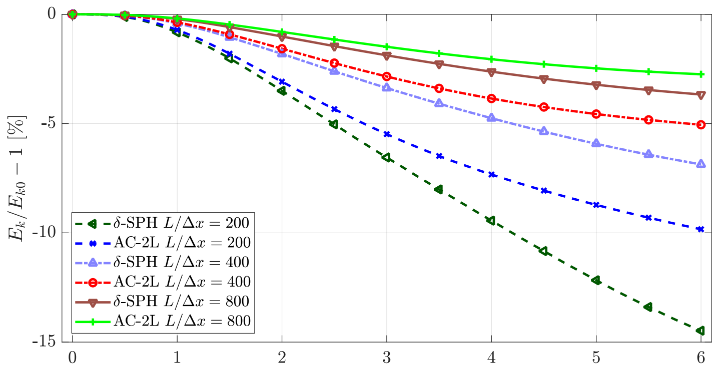




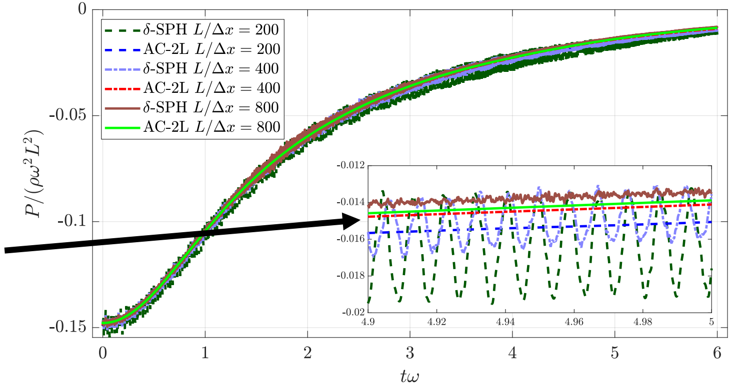


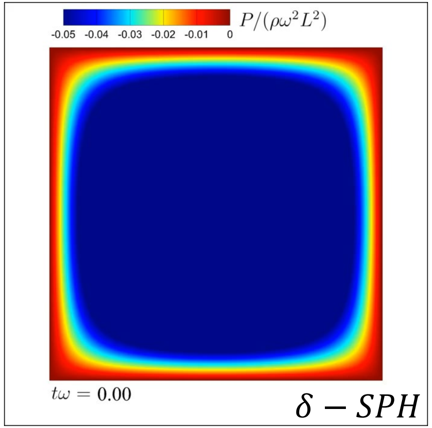
Case2: Rotation of a square patch

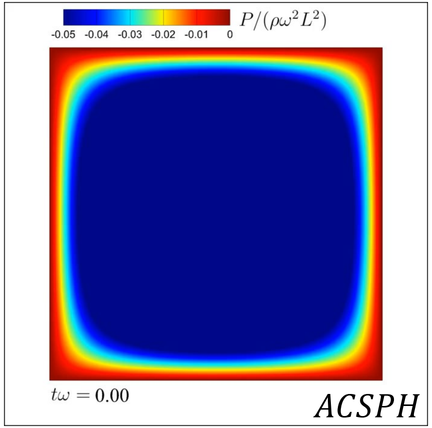
- Large deformations of the free surface with negative pressures.
- Strongly dependent on particle re-meshing or TIC approaches to avoid fragmentation.



- Very similar kinematics of the flow in AC and δ-SPH.
- Notably improved pressures recorded in the ACSPH scheme and apparent improvements in conservation properties.
- Smooth pressure even at coarsest resolution.



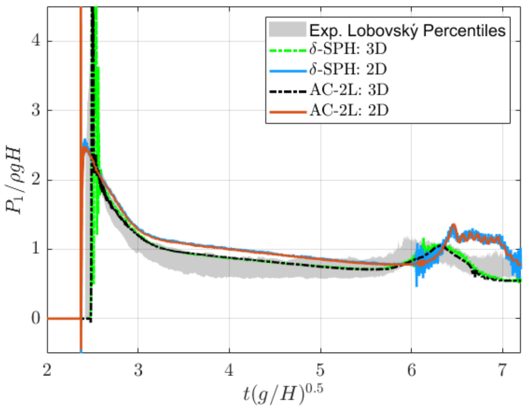


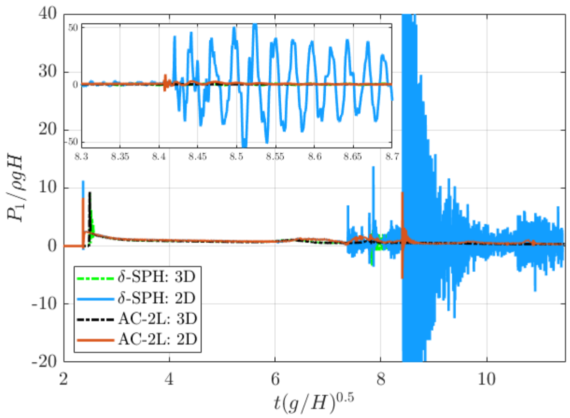


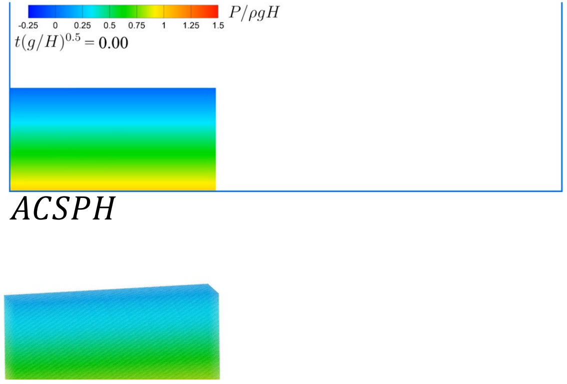
Axes normalised by time to account for spin-out

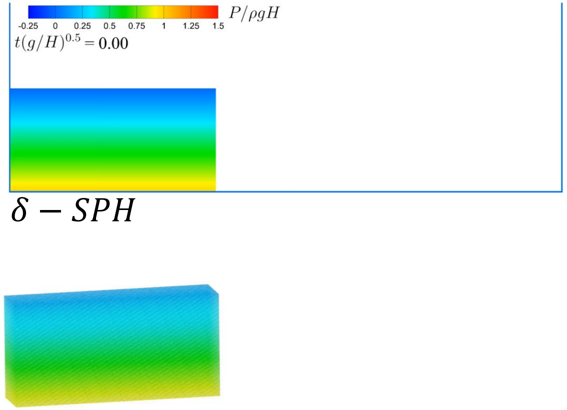
Case 3: Dam break

- Iterative nature of ACSPH scheme ensures pressure boundary conditions are resolved on the first iteration, whereas WCSPH resolves this in time
- Initial fluid impact shows similar pressure profiles between AC and WCSPH, with reduced noise in ACSPH; subsequent fluid interaction induces strong pressure oscillations in WCSPH from acoustic waves



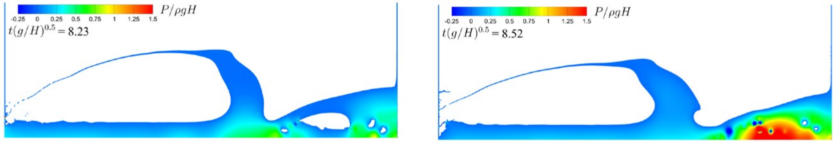




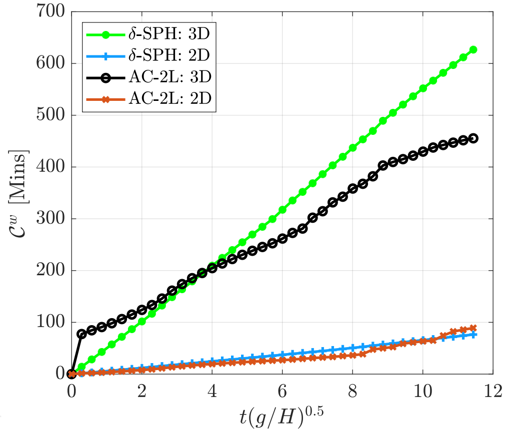


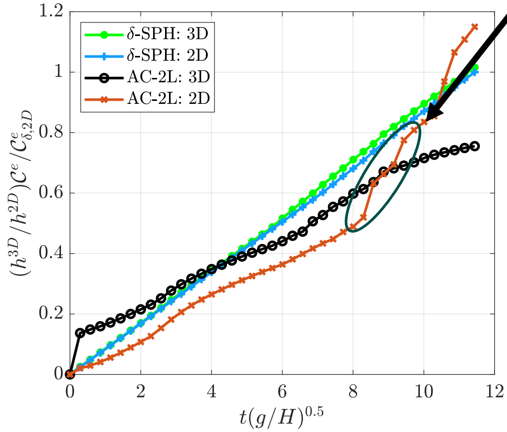
Case 3: Dam break (Cost)

- Requirement for iteration in ACSPH implies a higher cost?
- This is a balance between the stability criterion
  - WCSPH $\Delta T \propto h/c_0$
  - ACSPH $\Delta T \propto h$
- And the number of iterations required to solve the scheme
- With correct parameterisation of the time-integration scheme, ACSPH can be as fast as δ-SPH.



ACSPH 'works' harder where it is required





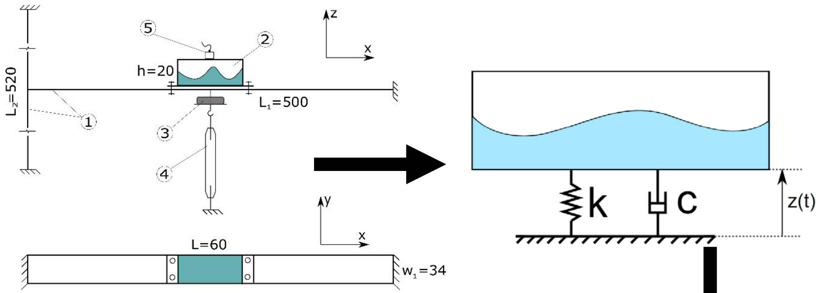
- $\mathcal{C}^e$ (right): 'Number of function evaluations required to complete the solution'. Removes question of software optimisation and hardware use

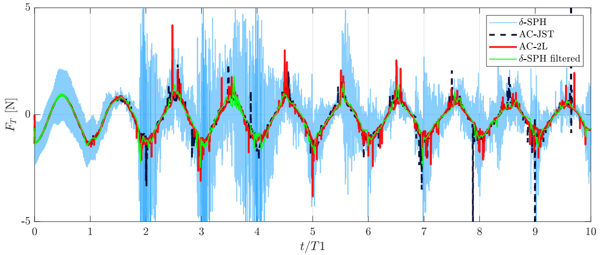
$$\mathcal{C}^e = \int_0^t \frac{m_{iter} s_{RK}}{\Delta t} dt$$

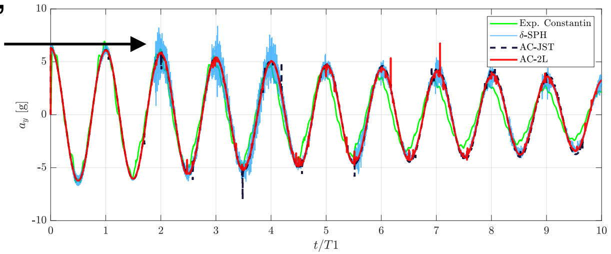
- $\mathcal{C}^w$ (left): Wall time (on 1 P100 GPU)

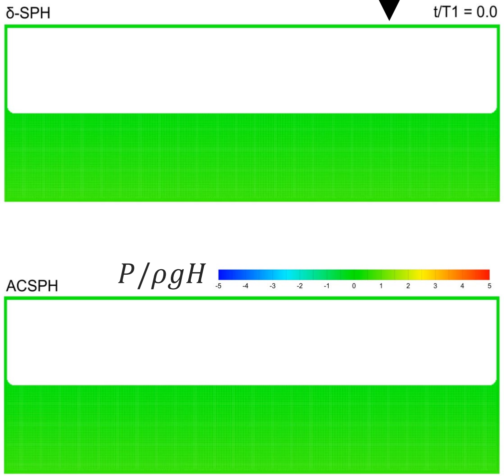
Case 4: Vertical sloshing

- Purely vertical excitation at 10.05 Hz and up to 7g acceleration
- WCSPH shows acoustic noise arising immediately from impulsive release and excitation
- This can be filtered out to recover the incompressible solution (as shown here)
- Filtering can not be applied in FSI cases; noise remains in structural accelerations
- Structures (with inertia) act as a low-pass filter, reducing the high-frequency noise here









Conclusions

- Weak compressibility can be problematic for some FSI cases. Acoustic effects appear in the calculated pressures/forces, and can manifest themselves in structural time histories. Forces from forced motion are hard to compare due to noise
- For our work, it was beneficial to move to full incompressibility, which was achieved with ACSPH. Or, this, can of course also be thought of as a pseudo-loop for the ISPH system, because…
- …ISPH, ACSPH and δ-SPH are *very* closely associated. There is even no obvious reason to prevent mixing them, if it were to be beneficial
- This means AC ideas can be mapped to δ-SPH and ISPH
- It may be possible to construct other pressure smoothing terms, as tested in the JST context here